Improving numerical efficiency with model reduction and high-order adaptive discontinuous Galerkin

Antonio Huerta
Laboratori de Càlcul Numèric (LaCàN), Dept. Applied Math.,
E.T.S. de Ingenieros de Caminos,
Universitat Politècnica de Catalunya-BarcelonaTech, Barcelona, Spain
Email: antonio.huerta@upc.edu

Abstract

Despite the impressive progress attained in the last decades by simulation-based engineering sciences, decision-making in engineering design, optimization and control, remains sub-optimal in many fields. Aerospace industry is probably one area where these limitations are more obvious.

On one hand, in a multitude of real engineering design problems a large number of scenarios must be considered and carefully analyzed. This task is very expensive both in specialized man-hours to prepare and analyze data and from a computational point of view. The space of design parameters is, in these cases, too large for an exhaustive exploration. In general, only a small sample of the parametric space is studied. Consequently, these models must be complemented with security coefficients conceived to cover the rest of the parametric space and to include unknown information, the inevitable uncertainty. Thus, in practice, designs remain suboptimal because of the computational complexity related to very rich descriptions of external actions, geometry, materials, processes, etc. In fact, even for cutting edge engineering, real practice imposes methodologies devised more than 30 years ago.

On the other hand, two contradictory goals are nowadays present in every challenging simulation based engineering problem: real-time and high fidelity. In order to speed-up engineering design or to assist decision-making strategies in engineering processes, faster simulations are required. Moreover, in many cases, there is the added restriction: such decision-making tools should run in light computing devices to increase portability, on-site evaluation, or democratize accessibility. Real-time is easier to attain with coarse models or meta-models involving few number of parameters. These requirements usually are contradictory with high-fidelity simulations. Moreover, users are more demanding and prescribe error bounds on quantities of interest to minimize uncertainty in models and simulations.

These apparently incompatible goals can be integrated by means of a *computational vademecum*. A high-fidelity error-controlled off-line computation produces a solution of the model under consideration for all the possible design scenarios. Then, an on-line post-process, able to run on light computational devices if necessary, is used for fast decision-making purposes.

The Proper Generalized Decomposition (PGD), which relies in the assumption of separated approximations of the solution, has demonstrated its capabilities in dealing with high-dimensional problems. The multidimensional capabilities of this approach opens new possibilities to solve, for instance, problems where material or external parameters are set as additional extra-coordinates of the model. In this framework a general solution is obtained encompassing every solution for any possible value of the parameters, thus, a computational vademecum is produced. Under this rationale, parametric design, optimization of complex problems, uncertainty quantification, simulation-based control, and real-time simulation are seen as a post-process once the off-line strategy has produced the vademecum.

To illustrate the advantages of such an approach a simple shape optimization example will be shown before discussing a practical engineering problem governed by the Helmholtz equation with variable coefficients in an unbounded domain. This problem models harbor wave agitation, which is a primary engineering design challenge. Two major issues are discussed:

1. *Efficient and accurate computations*. This implies a large number of simulation challenges, which include, among others, reproducing the exact geometry to capture the small features that are influential, efficient adaptive approximations, precise high-gradients (shock-capturing) approximations, etc.

2. *Large number of external forcing conditions*. A general solution for the agitation in the harbor is obtained with the incident wavelength and its direction as extra coordinates covering an exhaustive evaluation of all possible scenarios and enabling on-line computations on tablets.